Price Dynamics When There Are Alternatives to Cash Payment

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Abstract

Empirical evidence from the 1980s and 1990s indicates that cash use in the U.S. remains high even though there has been a proliferation of alternatives to cash. This paper examines the dynamics of inflation and asset prices in response to innovations in the efficiency of processing non-cash transactions. The quantitative results suggest that inflation is more sensitive than nominal interest rates or real equity prices to innovations in the efficiency of non-cash payments processing. Thus, as alternatives to cash payment become more prominent, the volatility of real interest rates may increase. (*JEL* E31, E41, G12)

Introduction

This paper evaluates the quantitative importance for inflation and asset prices of the presence of alternatives to cash payment. Hancock and Humphrey (1998) document that non-cash transactions per person in the U.S. rose by 58.7 percent between the 1980s and 1990s. They attribute this increase to advances in payments technology and a broad-based expansion in the availability of alternatives to cash (such as point-of-sale terminals). They also report, however, that cash use in the U.S. remains high. In fact, the U.S. was one of only a few countries where the cash-to-GDP ratio increased from the 1980s to the 1990s.[1] Hancock and Humphrey interpret this measure (along with the value of cash holdings per person) as a proxy for the intensity of cash use for transaction purposes.

Figures 1 and 2 present time series evidence on real home currency balances per person and the velocity of home currency from 1980 through 1998. This time span covers a period where a substantial amount of innovation in payment technology occurred. Along with fluctuations in interest rates, these innovations no doubt account for some of the fluctuations observed in the figures. It does not appear, however, that currency is being completely driven out of the system. In

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[1] Their finding is not a result of using estimates of the U.S. currency stock inflated by U.S. currency held abroad. They adjust their currency figures using estimates of U.S. currency held abroad by Porter and Judson (1996). Jefferson (2000, 1998) and Porter and Weinbach (1999) illustrate how failure to make this correction may lead to misleading inferences.

fact, the trend in velocity is, if anything, slightly negative. Post-1982, the trend in real currency balances per person is, if anything, slightly positive.[2] Table 1 presents some summary statistics on velocity and real home currency balances. Together with the Hancock and Humphrey evidence, these findings suggest that like its industrialized counterparts, the U.S. is in the process of developing its own societal-specific alternatives to cash payment. At the same time, it does not appear that cash is going anywhere anytime soon.

FIGURE 1. REAL HOME CURRENCY BALANCES PER CAPITA

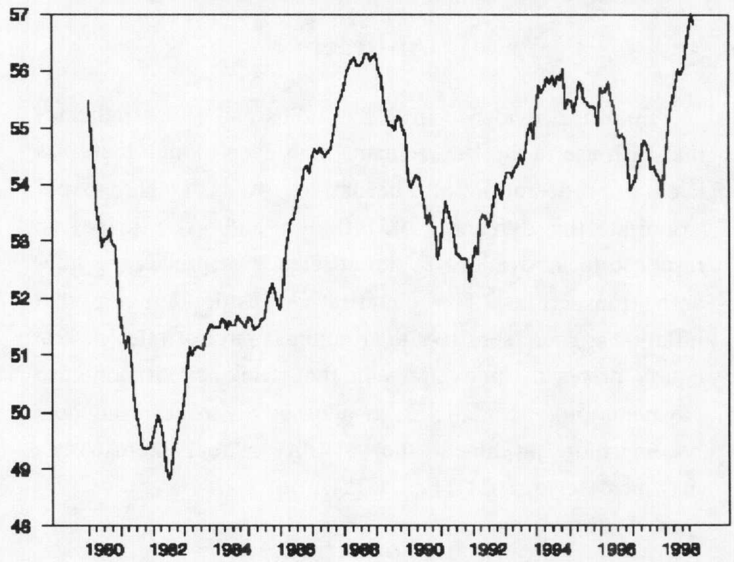


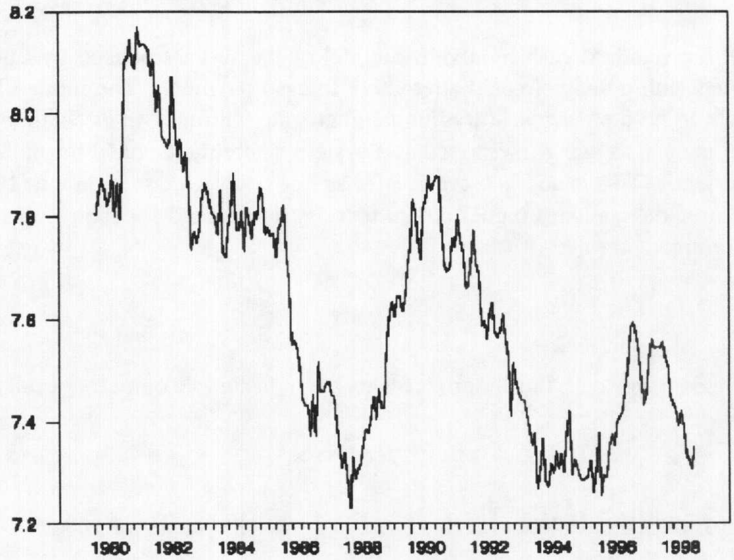
TABLE 1. BASIC STATISTICS 1980:1 – 1998:12

	Velocity	Currency Balances
Mean	7.619	53.600
Standard Deviation	0.238	1.975
Trend	-0.003	0.021
	(0.0002)	(0.001)

Notes: The data are monthly. Velocity is measured as the ratio of personal consumption expenditure of nondurable goods to home currency. Home currency is total U.S. currency held outside banks minus U.S. currency held abroad. Currency balances are home currency deflated by the consumer price index and population (16yrs +). The unit of currency balances is real dollars per month. Trend is the coefficient on time from a simple OLS regression of the given variable on time and a constant. Standard errors are in parentheses.

[2] These statements are not meant to imply that these trends have been stable (or that they are not simply capturing the trend behavior of nominal interest rates) over the full period for which we have comparable data. For example, velocity rose and real per capita home balances fell throughout the 1970s. Rather, they attempt to characterize broad movements in these variables in the post personal computer period—a period in which advancement in the development of alternatives to cash payment is thought to have been significant in principle if not in actual practice.

FIGURE 2. VELOCITY OF HOME CURRENCY



The development of alternatives to cash payment that are completely outside of traditional measures of money raises issues and concerns for policymakers. Included among these are the determination and variability of inflation and market interest rates, the evolution of currency balances, and the response of asset prices to technological innovations that enhance the usefulness of alternatives to cash payment.

In this paper, we present a modified cash-in-advance model to address these issues. The modification is the presence of an alternative stochastic technology for acquiring the consumption good. The presence of goods acquired in this way permits the model to be interpreted using the familiar credit/cash good (Lucas and Stokey 1983) or the market/home good (Scheinkman 1980) distinction. This technology is buffeted by technological disturbances that affect the efficiency with which it delivers the consumption good. The specification is similar in spirit to that used by Ireland (1995) except that, in his model, capital was durable. Here the focus is more on portfolio allocation considerations that permit abstraction from durable capital. Intrasectoral allocations of financial capital and the technological efficiency of the alternative to cash payment are reflected in asset prices. For example, one way for households to transfer currency into the future is by purchasing bonds today. Before surrendering today's currency holdings, however, households contemplate how they will acquire their consumption goods next period. If the alternatives to cash payment are expected to be highly efficient next period, then households are more willing to lend currency via bond purchases today. In equilibrium, this consideration links the nominal interest rate directly to the efficiency of alternatives to cash payment.

The remainder of the paper is organized as follows. The second section describes the model and its equilibrium. Asset prices for currency, nominal bonds, and equities in the presence of alternatives to cash payment are derived in the third section. In the fourth section, the model is linearized in order to examine the dynamic responses of the model's endogenous variables to innovations in income, money growth, and the technical efficiency of the non-cash sector. One of the main quantitative results in this section is that inflation is more sensitive than asset prices to innovations in the efficiency of non-cash payments processing. The final section is the conclusion.

The Model

The model is a standard cash-in-advance model of the sort introduced by Lucas and Stokey (1983) augmented with a technological alternative to cash payment.[3] The money in the model is explicitly thought to be the narrowest traditional aggregate: currency in circulation outside banks. Thus, the model is open to being interpreted as a system where the demand for money has whittled away to only currency. This is not, however, as far as one could go, as Cochrane (1999) shows in a modified version of the standard model where there is no demand for money at the end of each period in equilibrium.

Firms

The final goods firm stochastically produces its real nonstorable output, y_t, each period.

Preferences

The economy consists of identical infinitely lived households. The representative household's preferences are

$$E_t \sum_{t=0}^{\infty} \beta^t u(x_t). \tag{1}$$

The utility function, u, is twice differentiable with $u' > 0$, $u'' < 0$, $u' \to 0$ as $x_t \to \infty$ and $u' \to \infty$ as $x_t \to 0$. The discount factor, β, satisfies $0 < \beta < 1$. Total household consumption, x_t, is the sum of consumption obtained via the use of currency x_t^c and consumption obtained via processing services, x_t^d. The relative price of x_t^c and x_t^d is one. The specific form for u employed in the quantitative analysis reported below is

$$u(x_t) = \frac{x_t^{1-\gamma} - 1}{1 - \gamma} \tag{2}$$

for $\gamma \in (0,1)$ and $u(x_t) = \log(x_t)$ for $\gamma = 1$.

Timing

Three assets and a particular timing of market transactions facilitate and influence households' decisions. Households enter period t with a predetermined amount of privately issued nominal bonds, B_t, claims (shares) to the firm, S_t, and currency, C_t. At the beginning of period t, the shopper leaves home with C_t. The firm's output, y_t, is realized and distributed to households according to $S_t y_t$. This constitutes the dividend session for period t. Assume that households are not able to directly consume their own dividend at any time. Period t's shopping session opens next. Shoppers demand goods with C_t. Contingent on their desired and actual asset positions, households sell all or part of $S_t y_t$ at price P_t. Households that do not sell all of $S_t y_t$ allocate the remainder of their wealth to the processing sector. This sector produces services that "process"

[3] The strengths and weaknesses of the standard cash-in-advance model have been explored by Hodrick et al. (1991), Christiano (1994) and Walsh (1998).

this flow of funds and returns consumption equivalents to households. This concludes the shopping session. Households then settle, according to $(1 + r_t^n)B_t$, their bond positions, and the monetary transfer (tax), H_{t+1}, occurs during the settlement session for period t. If positive, the transfers cannot be used to purchase consumption in the current period. Next, shares, S_{t+1}, are traded for currency at price Q_t, and bonds, B_{t+1} are positioned for currency during period t's security trading session. Finally, the shopper and seller return home.

The shopper's activities are constrained by the amount of currency brought into the period. Therefore,

$$P_t x_t^c \leq C_t \tag{3}$$

The seller decides how much of the household's dividend to sell in the final goods market. Each household's supply is atomistic relative to the market. Therefore, each household takes P_t, Q_t, and r_t^n as given. The household supplies its entire dividend to the goods market if

$$P_t S_t y_t \leq C_{t+1} + B_{t+1} + Q_t[S_{t+1} - S_t] - H_{t+1} - (1 + r_t^n)B_t \tag{4}$$

Inequality (4) assumes that information flows freely across markets and between agents even though markets are temporally separated and household members are spatially separated. Inequality (4) holds when the market value of current household dividends is low. In this case, the household does not receive enough currency in period t to finance its desired asset position for the next period. Therefore, the shopper behaves so that the CIA constraint does not bind, thereby providing the household with more cash. In this case, the household maximizes (1) subject to the constraint

$$P_t x_t^c + C_{t+1} + B_{t+1} + Q_t S_{t+1} = C_t + S_t[P_t y_t + Q_t] + H_{t+1} + (1 + r_t^n)B_t. \tag{5}$$

If inequality (4) is reversed, then the household is awash in cash. In this case, the shopper spends all currency carried into the period so that the CIA constraint binds and the seller sells only a portion of the dividend thereby accumulating funds for transfer to the processing sector. Denote these funds as

$$k_t^d \equiv S_t y_t - (C_{t+1} + B_{t+1} + Q_t[S_{t+1} - S_t] - H_{t+1} - (1 + r_t^n)B_t) / P_t. \tag{6}$$

Processing: The Alternative to Cash

Let the processing technology be governed by the function $\theta_t g(k_t^d)$. The processing shock, θ_t, is strictly positive and stationary with mean one. The specific form for g employed in the quantitative analysis reported below is

$$g(k_t^d) = \frac{(1 + k_t^d)^{1-\alpha} - 1}{1 - \alpha} \tag{7}$$

for $\alpha \in (0,1)$.

Clearly, such a specification is an abstraction that is meant to represent in a simple way the technology-based alternatives to cash payment highlighted by Hancock and Humphrey (1998). These alternatives cover an ever evolving range from refinements of electronic funds transfer (EFT) for large payments to the development of cryptography that should enhance the implementation of electronic cash systems for small denomination transactions. In the context of the model, this technology delivers the consumption good without the use of cash. There is precedent for a specification of this type. In Ireland (1995), for example, the distance from home that a household can use credit to acquire the consumption good is determined by a production function representing the state of payments development. Earlier, King and Plosser (1984) modeled an intermediate transactions sector in a similar way. The output from their transactions sector was then used in the production of the consumption good. Even earlier, Scheinkman (1980) introduced an alternative to cash in a nonstochastic monetary model with home production. In either case, the essential feature is the presence of an alternative process for acquiring the consumption good, be it viewed as the credit, intermediate, or home good. In all of these models, investment of time or resources is required in order to activate the alternative to cash. That tradition is preserved here by the requirement that financial capital, k_t^d, be positive in order to acquire the consumption good via the processing technology. The stochastic nature of the technology captures possible randomness in the efficiency of processing.

Household consumption acquired through the services provided by the processing sector is constrained by

$$x_t^d = \theta_t g(k_t^d). \tag{8}$$

When consumption accrues to the household as a result of processing services and currency transactions, the household maximizes (1) subject to the constraints (3), (8), and

$$x_t = C_t / P_t + \theta_t g(k_t^d). \tag{9}$$

The supply of currency, C_t, evolves according to

$$C_{t+1} = C_t + H_{t+1}$$

where $H_{t+1} = \sigma_t C_t$ and σ_t (the growth rate) is stochastic.

Household's Problem and FOC's

Define $\Omega_t = \{\theta_t, y_t, \sigma_t\}$ as the state of the system at time t. In addition to the goods market and monetary shocks that are usually represented, Ω_t has an additional term, θ_t, that attempts to capture the impact of randomness in the efficiency of processing. As in Svensson (1985), the timing convention for the state is that Ω_t is revealed at the beginning of period t. A solution to the household's problem is given by

$$V(S_t, B_t, C_t, \Omega_t) = \max\{u(x_t) + \beta E_t V(S_{t+1}, B_{t+1}, C_{t+1,} \Omega_{t+1})\}$$

subject to the constraint (5) if $k_t^d \leq 0$; the constraints (3), (8), and (9) if $k_t^d > 0$, and (6). Consider the function $f_t(k_t^d)$ with $f_t(k_t^d) = k_t^d$ if $k_t^d \leq 0$ and $f_t(k_t^d) = \theta_t g(k_t^d)$ if $k_t^d > 0$. Then, for all k_t^d

$$x_t = C_t / P_t + f(k_t^d) \tag{10}$$

Equation (10) combines constraints (5) and (9).[4]

If attention is restricted to interior stationary solutions for which the function V exists, is differentiable, and concave, then the first order conditions for the household's optimal choices of $C_{t+1,} B_{t+1}$, and S_{t+1} are

$$u'(x_t) f_t' / P_t = \beta E_t[u'(x_{t+1}) / P_{t+1}] \tag{11}$$

[4] This construct for combining constraints follows Scheinkman (1980).

$$u'(x_t)f_t' / P_t = \beta(1 + r_{t+1}^n)E_t[u'(x_{t+1})\, f_{t+1}' / P_{t+1}] \tag{12}$$

$$u'(x_t)f_t' q_t = \beta E_t[u'(x_{t+1})\, f_{t+1}'(y_{t+1} + q_{t+1})] \tag{13}$$

In these equations, if $k_t^d > 0$, $f_t' = \theta_t g'(k_t^d)$ is the marginal productivity of the processing technology. Otherwise, $f_t' = 1$ because equation (5) is the constraint. The real stock price is $q_t \equiv Q_t / P_t$.[5]

These equations suggest the following intuitions. Equation (11) says that the household holds currency until the marginal utility cost of forgoing a unit of consumption (acquired via currency transactions or processing services) today and holding an additional dollar is equal to the expected discounted marginal utility benefit of having a dollar next period for the acquisition of a unit of consumption goods via currency transactions. Equation (12) equates that same marginal utility cost to the expected discounted utility value of investing a dollar in a bond today and having the return from that investment available next period for consumption acquired via currency transactions or processing services. Equation (13) suggests that shares are held until the marginal utility cost of holding an additional dollar's worth of shares equals the expected marginal utility value of consumption (acquired via currency transactions or processing services) from future dividend and asset sales. Finally, let μ_t, ϕ_t, λ_t^0 and λ_t^1 be the multipliers associated with the constraints (3), (8), (5), and (9), respectively. Then, $u'(x_t) = \lambda_t^0$ when only currency is used. When currency and processing services are used, $u'(x_t) = \mu_t + \lambda_t^1 = \phi_t + \lambda_t^1$ which implies $\mu_t = \phi_t$; consumption occurs until the value of an additional unit of currency and processing services is equal.

Equilibrium

An equilibrium for this model is defined as follows. Given the stochastic process $\{\Omega_t\}$, a monetary equilibrium is the nonnegative processes $\{r_t^n, Q_t\}$, the positive sequence $\{P_t\}$, the choice processes $\{S_t, B_t, C_t^{demand}\}$, and the function V that solve the household's problem and cause the bond, stock, and goods markets to clear at each time t. That is, $B_t = 0$, $S_t = 1$, and $x_t = C_t / P_t + f(k_t^d)$.[6]

Analytical Results

Prices in the Presence of Cash Alternatives

In a stationary rational expectations equilibrium, the first order conditions can be used to derive asset pricing relations for currency, bonds, and stocks. For currency, iteration on (11) yields

$$1/P_t = (1/\lambda_t f_t')E_t\left[\prod_{j=1}^{\infty}(\beta / f_{t+j}')\lambda_{t+j+1} / P_{t+j+1}\right] \tag{14}$$

[5] The transversality conditions are $\lim_{T\to\infty} E_t\beta^{T-1}u'(x_{t+T-1})f'_{t+T-1}C_{t+T} / P_{t+T-1} = 0$, $\lim_{T\to\infty} E_t\beta^{T-1}u'(x_{t+T-1}) f'_{t+T-1}B_{t+T} / P_{t+T-1} = 0$, and $\lim_{T\to\infty} E_t\beta^{T-1}u'(x_{t+T-1})f'_{t+T-1}q_{t+T-1}S_{t+T} = 0$.

[6] In Appendix A, equilibrium in the special case where Ω_{t+1} is independent of Ω_t is considered in detail. In this case, the price level is more responsive to dividend and monetary shocks when an alternative to cash payment is available relative to a regime where only cash is used for transactions. The robustness of this result, however, is sensitive to the degree of relative risk aversion.

where $\lambda_t = u'(x_t)$ is the multiplier associated with (10). Substituting (11) into (12) and using the definition of the conditional covariance yields

$$(1+r_{t+1}^n)^{-1} = E_t[f'_{t+1}] + Cov_t[\lambda_{t+1}/P_{t+1}, f'_{t+1}]/E_t[\lambda_{t+1}/P_{t+1}] \tag{15}$$

Finally, iteration on (13) yields

$$q_t = (1/\lambda_t f'_t)E_t\left[\sum_{j=1}^{\infty}\beta^j \lambda_{t+j} f'_{t+j} y_{t+j}\right] \tag{16}$$

Equation (14) suggests that the price of currency depends on current and all future marginal products of the processing services technology. If currency is valued in the current and all subsequent periods, then there must be some periods $s > t$ where households expect $f' > \beta$. If this is not the case, then the current value of currency is arbitrarily large relative to its future value. Correspondingly, if households expect $f' > \beta$ for all $s > t$, then the current value of currency is arbitrarily low relative to its future value. The reason is that the processing sector is (and is expected to be) too productive to support a demand for currency.

The bond price equals the expected marginal product of the processing sector plus a term that depends on the covariance between the marginal product of the processing sector and the marginal utility of a dollar's worth of consumption acquired via currency transactions. Some intuition for why this is so may be gained by considering each component separately. First, ignore the covariance term. The household can acquire the good next period using currency carried over into the next period or via processing services. If the marginal product of the processing sector is expected to be low next period, then the household is less willing to lend its current cash via bond purchases today. The demand for bonds therefore decreases today while more bonds are issued. Bond prices fall and, as indicated by equation (15), nominal interest rates increase.

Now consider the covariance term. This additional term has a natural interpretation as a risk premium. When $Cov_t[\lambda_{t+1} / P_{t+1}, f'_{t+1}] > 0$, processing services are very productive when the marginal utility of a dollar's worth of consumption is high and less productive when the marginal utility of a dollar's worth of consumption is low. Risk averse agents are more willing to lend their currency in this case because they know that, when the marginal utility of a dollar's worth of consumption is high, processing services efficiently deliver consumption and, when processing services are not very efficient, they have a relatively low valuation of extra consumption.

Conversely, when $Cov_t[\lambda_{t+1} / P_{t+1}, f'_{t+1}] < 0$, risk-averse agents are less willing to lend currency because, when they value extra consumption highly, processing services are relatively inefficient in delivering consumption. When processing services are relatively efficient, they have a low valuation of extra consumption. Risk-averse agents, therefore, will want to hold (not lend) their currency so that they can acquire the good when their valuation of consumption is high.

This behavior toward risk is aptly characterized as a precautionary demand for currency. It arises endogenously even though the CIA constraint is binding whenever households use processing services. In this economy, equation (15) suggests that optimizing agents behave so that short-term nominal bond prices are a risk-corrected measure of the expected marginal product of processing services.[7]

Equation (16) indicates how processing services affect share prices. It takes into account that future dividends may be converted into currency or processing services and that the future

[7] Equation (15) also implies that a positive nominal interest rate is associated with a positive expected flow of funds to the processing sector next period.

productivity of processing services matters for the valuation of future dividends. The current share price is the expected discounted sum of all such future dividends. Holding current productivity constant, knowledge of higher future productivity in the processing sector increases the demand for claims to future production. This pressure forces the real value of these real claims to increase.

Premia: An Extension

By not restricting the flow of funds to one sector, the model generates a risk premium that induces a precautionary demand for currency. Processing shocks appear to be important for this premium. This finding raises the question of whether such shocks could be important for other premia that are present in traditional asset pricing models. We investigate this issue by writing down expressions for expected premia vis-a-vis an indexed bond and the equity premium for this model.[8]

The return on an indexed bond is $(1 + r^i_{t+1}) = \lambda_t f'_t / \beta E_t[\lambda_{t+1} f'_{t+1}]$. Let $R^s_{t+1} = (y_{t+1} + q_{t+1}) / q_t$. Using (13) yields the risk premium for stocks

$$E_t(R^s_{t+1}) - (1 + r^i_{t+1}) = \frac{-Cov_t[\lambda_{t+1} f'_{t+1} / \lambda_t f'_t, R^s_{t+1}]}{E_t[\lambda_{t+1} f'_{t+1} / \lambda_t f'_t]} .$$

Letting $R^n_{t+1} = (1 + r^n_{t+1})P_t / P_{t+1}$ and using (12) yields the risk premium for nominal bonds

$$E_t(R^n_{t+1}) - (1 + r^i_{t+1}) = \frac{-(1 + r^n_{t+1})Cov_t[\lambda_{t+1} f'_{t+1} / \lambda_t f'_t, P_t / P_{t+1}]}{E_t[\lambda_{t+1} f'_{t+1} / \lambda_t f'_t]}$$

Finally, the expected equity premium, $E_t(R^s_{t+1} - R^n_{t+1})$, is

$$\frac{-Cov_t[\lambda_{t+1} f'_{t+1} / \lambda_t f'_t, R^s_{t+1}] + (1 + r^n_{t+1})Cov_t[\lambda_{t+1} \; f'_{t+1} / \lambda_t f'_t, P_t / P_{t+1}]}{E_t[\lambda_{t+1} / \lambda_t]E_t[f'_{t+1} / f'_t] + Cov_t[\lambda_{t+1} / \lambda_t, f'_{t+1} / f'_t]} .$$

These expressions suggest that the usual marginal rates of substitution and returns now interact with the intertemporal ratio of processing sector productivity in the determination of risk premia. These interactions could lead to complex premia dynamics and may suggest that efforts to generate moments that match actual data moments using standard one-sector models may have been confounded by specification errors. Kocherlakota (1996) details the challenges posed by the equity premium.[9]

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[8] The equity premium and its solution are not the immediate focus here. These results, however, do indicate the relationship between Mehra and Prescott's (1985) finding and proposed solutions of, *inter alios*, Constantinidies (1990), Cecchetti, Lam, and Mark (1993), and Campbell and Cochrane (1999), and the present model.

[9] Townsend (1987) was among the first to show how a particular friction, spatial separation, that supported the valuation of money affected standard asset pricing formulas. More recently, Laker and Schreft (1996) demonstrated how costly credit that drives a wedge between the value of traded claims and the value of consumption altered standard asset pricing formulas.

Numerical Results

The Linearized Model

Further characterization of the equilibrium dynamics implied by the model is greatly facilitated by employing the specific functional forms given above for utility (equation 2) and the processing technology (equation 7). We can then explicitly characterize the steady state as in Table 2. We then compute the response of the model's endogenous variables to dividend, processing, and money growth rate shocks as percent deviations from this steady state.[10]

TABLE 2. STEADY STATE VALUES

Symbol	Value
x^{ss}	$c^{ss} + \theta^{ss}[(z^{ss})^{1-\alpha} - 1]/[1-\alpha]$
c^{ss}	$(1 + y^{ss}) - [\theta^{ss}(1 + \sigma^{ss})/\beta]^{1/\alpha}$
π^{ss}	σ^{ss}
y^{ss}	1.51
θ^{ss}	1
σ^{ss}	0.004
r^n_{ss}	$[(1 + \sigma^{ss})/\beta] - 1$
q^{ss}	$(\beta y^{ss})/(1-\beta)$
v^{ss}	x^{ss}/c^{ss}

The linearized model consists of exogenous processes for the dividend, processing, and money growth rate shocks plus the following five equations that can be solved for real currency balances, $\hat{c}_t$, the nominal interest rate, $\hat{r}^n_{t+1}$, the real stock price, $\hat{q}_t$, total consumption, $\hat{x}_t$, and inflation, $\hat{\pi}^n_{t+1}$:[11]

$$-\gamma\hat{x}_t + \hat{\theta}_t - \alpha\left(\frac{y^{ss}}{z^{ss}}\right)\hat{y}_t + \alpha\left(\frac{c^{ss}}{z^{ss}}\right)\hat{c}_t = E_t\left[-\gamma\hat{x}_{t+1} - \left(\frac{\pi^{ss}}{\omega^{ss}}\right)\hat{\pi}_{t+1}\right] \quad (17)$$

$$\hat{r}^n_{t+1} = \gamma\left(\frac{n^{ss}}{r^n_{ss}}\right)[E_t\hat{x}_{t+1} - \hat{x}_t] - \left(\frac{n^{ss}}{r^n_{ss}}\right)[E_t\hat{\theta}_{t+1} - \hat{\theta}_t] + \alpha\left(\frac{y^{ss}}{z^{ss}}\right)\left(\frac{n^{ss}}{r^n_{ss}}\right)[E_t\hat{y}_{t+1} - \hat{y}_t] -$$
$$\alpha\left(\frac{c^{ss}}{z^{ss}}\right)\left(\frac{n^{ss}}{r^n_{ss}}\right)[E_t\hat{c}_{t+1} - \hat{c}_t] + \left(\frac{\pi^{ss}}{\omega^{ss}}\right)\left(\frac{n^{ss}}{r^n_{ss}}\right) E_t\hat{\pi}_{t+1} \quad (18)$$

[10] In the equations that follow, constants super (or sub) scripted with *ss* are the steady state values of a variable. Additionally, a caret, ^, above a variable indicates that it is a percent deviation from its steady state value.

[11] The constants z^{ss}, w^{ss}, n^{ss}, and m^{ss} are defined as $z^{ss} = 1 + y^{ss} - c^{ss}$, $w^{ss} = 1 + \pi^{ss}$, $n^{ss} = 1 + r^n_{ss}$, and $m^{ss} = 1 + \sigma^{ss}$.

$$\hat{q}_t = -\gamma\ [E_t\hat{x}_{t+1} - \hat{x}_t] + [E_t\hat{\theta}_{t+1} - \hat{\theta}_t] - \alpha\left(\frac{y^{ss}}{z^{ss}}\right)[E_t\hat{y}_{t+1} - \hat{y}_t] + \alpha\left(\frac{c^{ss}}{z^{ss}}\right)[E_t\hat{c}_{t+1} - \hat{c}_t] + \left(\frac{y^{ss}}{y^{ss}+q^{ss}}\right) E_t\hat{y}_{t+1} + \left(\frac{y^{ss}}{y^{ss}+q^{ss}}\right) E_t\hat{q}_{t+1} \tag{19}$$

$$\hat{x}_t = \left(\frac{c^{ss}}{x^{ss}}\right)\left[1-\theta^{ss}\left(z^{ss}\right)^{-\alpha}\right]\hat{c}_t + \left(\frac{\theta^{ss}}{x^{ss}}\right)\left(z^{ss}\right)^{-\alpha} y^{ss}\hat{y}_t + \left(\frac{\theta^{ss}}{x^{ss}}\right)\left[\frac{\left(z^{ss}\right)^{1-\alpha}-1}{1-\alpha}\right]\hat{\theta}_t \tag{20}$$

$$\hat{\pi}_{t+1} = \left(\frac{w^{ss}}{\pi^{ss}}\right)\left(\frac{\sigma^{ss}}{m^{ss}}\right)\hat{\sigma}_t - \left(\frac{w^{ss}}{\pi^{ss}}\right)[\hat{c}_{t+1} - \hat{c}_t] \tag{21}$$

Equations (17), (18), and (19) are linear approximations to the first order conditions for currency, bonds, and stocks given earlier. Equation (20) is the linear approximation to the combined constraint equation (10). Equation (21) is derived from the fact that $c_{t+1} = [(1+\sigma_t)/(1+\pi_{t+1})]c_t$. Finally, equations (22), (23), and (24) describe the evolution of dividends, $\hat{y}_{t+1}$, technical change in processing, $\hat{\theta}_{t+1}$, and the money growth rate, $\hat{\sigma}_{t+1}$, respectively.

$$\hat{y}_{t+1} = \rho_y\hat{y}_t + e^y_{t+1} \tag{22}$$

$$\hat{\theta}_{t+1} = \rho_\theta\hat{\theta}_t + e^\theta_{t+1} \tag{23}$$

$$\hat{\sigma}_{t+1} = \rho_\sigma\hat{\sigma}_t + e^\sigma_{t+1} \tag{24}$$

These processes are stationary; thus the autocorrelation coefficients are less than one in absolute value. Additionally, the innovations, the ε's, are white noise with mean zero with variances Σ_y, Σ_θ, and Σ_σ, respectively.

Equations (17) through (24) constitute a linear rational expectations model that may be solved using a number of methods. We solved the model using the method of undetermined coefficients. The solution is such that each of the endogenous variables is a function of all of the exogenous variables. Details of these calculations are available from the author upon request.

Dynamic Responses

Table 3 shows the parameter values used in the calibration of the model. The parameter values for γ and β are standard. The parameter values for ρ_y, Σ_y, and y^{ss} (for dividends) and ρ_σ, Σ_σ, and σ^{ss} (for currency growth) are estimated from monthly data on real personal income per capita and home currency growth, respectively.[12] The estimations employ monthly data covering the period January 1980 through December 1998. As is usually the case for multiplicative technology shocks, we have $\theta^{ss} = 1$. We do not have direct estimates for the processing technology parameters. Indirect evidence, however, may be gleaned from Hancock et al.'s (1999) study of the

[12] The home currency figures are consistent with those constructed by Porter and Weinbach (1999).

Federal Reserve's electronic funds transfer operation. They report that for the period 1979 to 1996, the rate of technical change in electronic funds transfer processing was not very far from the average, national, private-sector productivity growth. This finding provides some evidence that starting with "standard" business cycle values as a benchmark for parameterizing technical progress in processing is, perhaps, not unreasonable. Nevertheless, absence of direct evidence implies that the parameters α, ρ_θ, and Σ_θ are essentially free.

TABLE 3. BASELINE PARAMETER VALUES

γ	α	ρ_θ	$\Sigma_\theta^{1/2}$	β	ρ_y	$\Sigma_y^{1/2}$	ρ_σ	$\Sigma_\sigma^{1/2}$
1	0.5	0.9	0.003	0.99	0.98	0.01	0.09	0.004

The steady state values of the model's endogenous variables implied by these parameter values are reported in Table 4. Velocity is higher than one as households make use of the processing technology.[13] Cash, however, is the dominant means by which the consumption good is acquired.[14] The monthly nominal interest rate is positive and somewhat high.[15] Finally, the real stock price is a significant multiple of dividends due to discounting.

TABLE 4. STEADY STATE AT BASELINE PARAMETER VALUES

Symbol	Value
x^{ss}	1.51
c^{ss}	1.50
π^{ss}	0.004
y^{ss}	1.51
θ^{ss}	1
σ^{ss}	0.004
r^n_{ss}	0.014
q^{ss}	149.49
v^{ss}	1.009

Figure 3 shows the responses of the model's endogenous variables to a one standard deviation shock to the processing technology. The positive shock implies that the processing technology is highly efficient in the current period and that it will be highly efficient in the near future before gradually returning to its steady state level. Agents respond to this by reducing their demand for currency. Their intrasectoral substitution away from currency in the period of the shock causes the price level to rise in the current period. The jump in the current price level causes the time period t

[13] By comparison, Table 1 indicates that average monthly velocity of home currency at an annual rate is 7.619 for the 1980-1998 period. The annual steady state velocity figure implied by the model is 12.108.

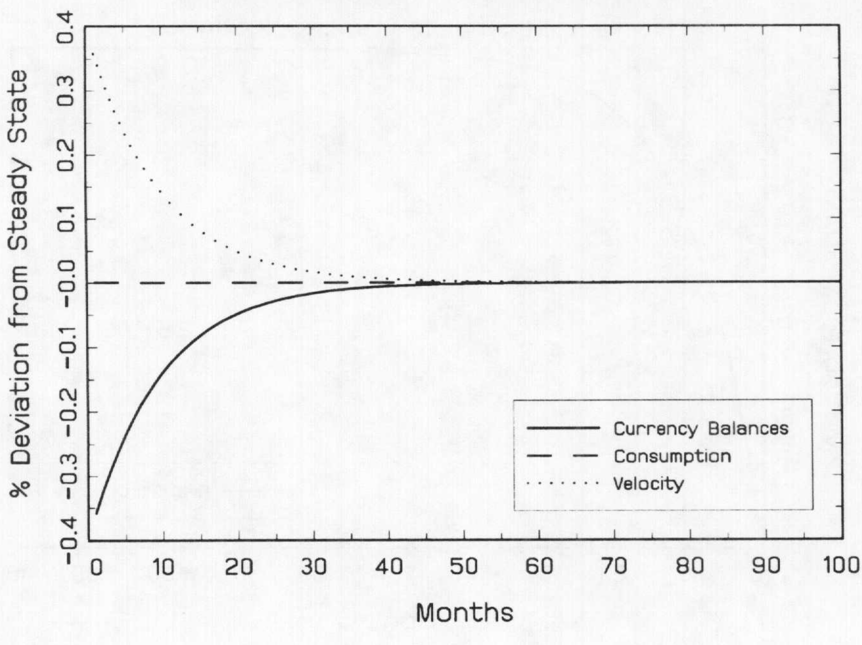
[14] Recall that the good in the model is nondurable. Thus, the closest real world analogue is nondurable goods. It may not be unreasonable to presume that cash transactions are the dominant means for acquiring this subset of total consumption.

[15] A monthly gross nominal rate of 1.014 corresponds to an annual gross nominal rate of approximately 1.182.

+ 1 inflation rate to fall. The nominal interest rate falls also but by less than the fall in inflation. In part, the reason for this is the anticipated decline in future processing efficiency created by the current shock, which will cause agents to be less willing to lend currency in the future. Thus, on impact, the implicit real interest rate rises. The real stock price falls slightly. Two offsetting factors are operating here. First, on impact, the shock induces agents to substitute away from equity in the current period. This combined with the rise in the price level depresses the real stock price. However, the facts that the processing technology will be highly efficient in the near future and that processing requires investment in order to have access to it increases the demand for equity in the firm in the current period. This is the second factor. The response shown is the net effect, which indicates that the first factor slightly outweighs the second. The impact of the shock on total consumption is positive but negligible. The shock is transitory, and agents adjust their portfolios so as to smooth consumption. The combined effect of the consumption and currency balance responses is an increase in velocity.[16]

Overall, households in the model use currency intensively in the purchase of nondurable goods. This is in part due to the fact that the average home currency growth rate at approximately 4.9 percent (at an annual rate) over the 1980-1998 period is quite moderate. Across the different shocks, inflation is the most sensitive variable. All of the shocks considered above are temporary. Of particular interest, however, is how the dynamic and stochastic structure of the economy might change as alternatives to cash payment gain prominence on average. We turn to this issue in the next section.

FIGURE 3A. RESPONSE TO A PROCESSING SHOCK



[16] The dynamic responses of the model's endogenous variables to dividend income and currency growth rate stocks are presented in Appendix B.

FIGURE 3B. RESPONSE TO A PROCESSING SHOCK

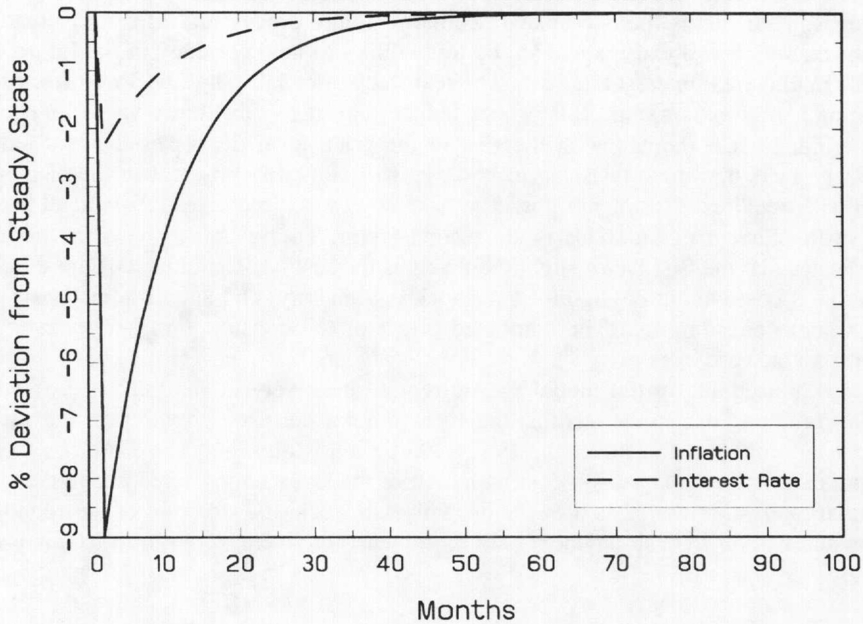
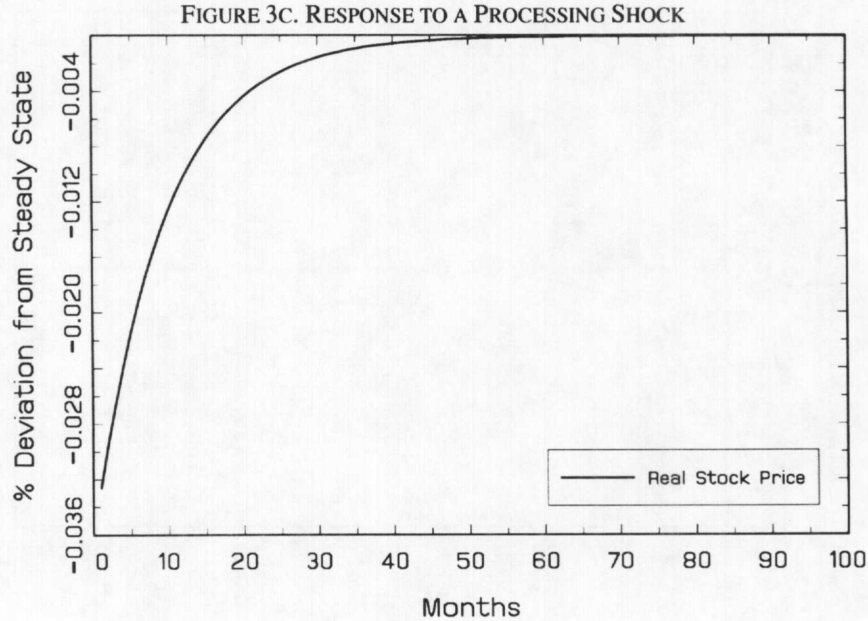


FIGURE 3C. RESPONSE TO A PROCESSING SHOCK



A Mean-Variance Relationship

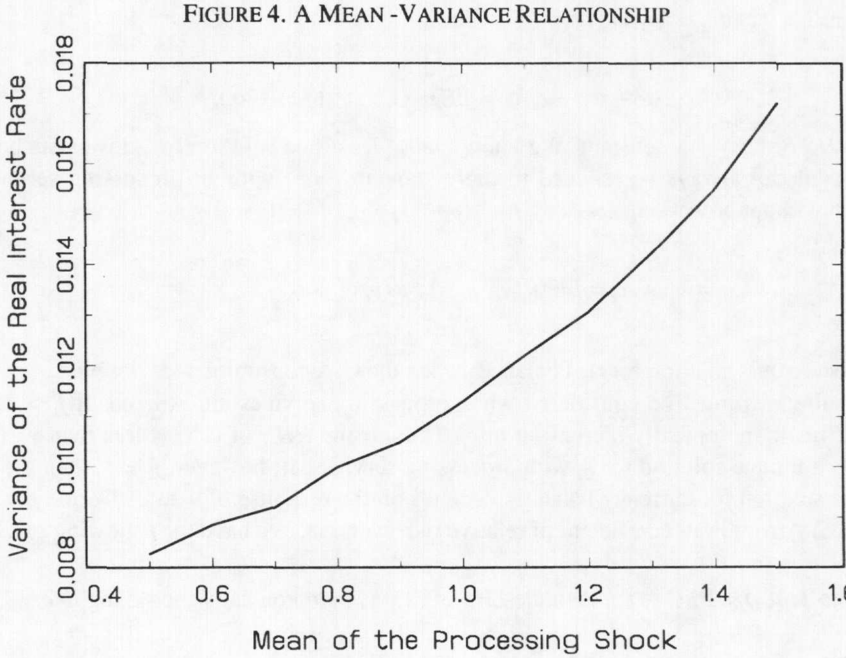
An increase in the importance of alternatives to cash payment may be approximated by changes in the average value of the processing sector technology shock. In this section, we consider the impact of such changes on the variability of real interest rates.

The real interest rate, τ, is implicitly defined within the model. Our treatment of it parallels directly that of the other endogenous variables. First, its steady state value is characterized. Then we linearize in terms of percent deviations around this steady state. Using the fact that $(1+\tau^{ss}) = (1+r^n_{ss})/(1+\pi^{ss})$, Table 2 indicates that $\tau^{ss} = \beta^{-1}-1$. It is straightforward, then, to show that[17]

$$\hat{\tau}_t = \left(\frac{l^{ss}}{\tau^{ss}}\right)\left(\frac{r^n_{ss}}{n^{ss}}\right)\hat{r}^n_{t+1} - \left(\frac{l^{ss}}{\tau^{ss}}\right)\left(\frac{\pi^{ss}}{\omega^{ss}}\right)\hat{\pi}_{t+1}$$

For each value of the mean of the processing sector technology shock, we simulate the linearized model several times. This yields a different variance of $\hat{\tau}_t$ for each value of the mean of the processing shock.[18] The relationship between the mean of the processing sector technology shock and variance of the real interest rate is monotonic and positive. Figure 4 shows the relationship for a subset of values for the mean of the processing sector technology shock over which the simulation was performed. The results in Figure 4 suggest that as alternatives to cash payment become more prominent, the volatility of real interest rates may increase.

FIGURE 4. A MEAN-VARIANCE RELATIONSHIP



[17] The constant l^{ss} is defined as $l^{ss} = 1 + \tau^{ss}$.

[18] Specifically, we construct 200 representations of the processing sector technology shock vector, $\hat{\theta}_t$ for each value of the mean. Each representation of $\hat{\theta}_t$ is then fed into the model yielding 200 representations of the real interest rate vector, $\hat{\tau}_t$. The variance of each representation of the $\hat{\tau}_t$ vector is computed. Then, the average of the 200 variances is taken to be the variance of $\hat{\tau}_t$ for that value of the mean the processing sector technology shock.

Conclusion

Currency use in the U.S. remains high even though alternatives to cash payment continue to proliferate. This fact presents a challenge to policymakers who seek to understand how the presence of alternatives to cash payment might affect the determination and dynamics of inflation and asset prices. The analytical results from an elementary cash-in-advance model indicate that the efficiency of transactions processing can have a significant role in the determination of assets prices due to households' substitution patterns across payment media. The quantitative importance for asset prices of the presence of alternatives to cash payment, however, varied substantially depending on the asset and the type of shock considered. For example, inflation is more sensitive than nominal interest rates or real equity prices to innovations in the efficiency of non-cash payments processing. Thus, as alternatives to cash payment proliferate, the volatility of real interest rates may increase. Our results also suggest that the success of efforts to target inflation may depend crucially on whether consideration of the payment options of households has been incorporated into the design of policy.

Appendix A

A Special Case. To gain some insight into the properties of an equilibrium, consider the special case where Ω_{t+1} is independent of Ω_t. In this case, the probability distribution of Ω_{t+1} is $G(\Omega_{t+1}; \Omega_t) = G(\Omega_{t+1})$. Writing equation (11) in real balance form and noting that in any equilibrium $B_t = 0$ and $S_t = 1$ yields

$$c_t u'(c_t + f(y_t - c_t))f_t' = \beta E_t[u'(x_{t+1})c_{t+1}] / (1+\sigma_t). \tag{25}$$

The *RHS* of (25) is a constant. We know that at least one solution for (25) exists because the present model can always be reduced to more standard ones without processing services. In an equilibrium without processing services, $f(y_t - c_t) = y_t - c_t$, $f' = 1$, and

$$c_t = \beta E_t[u'(y_{t+1})c_{t+1}] / u'(y_t)(1+\sigma_t) \tag{26}$$

which is Svensson's solution for real balances when the CIA constraint does not bind.

Two sub-cases must be considered when processing services are utilized. If $f_t' > 1$, then the *LHS* of (25) is monotonically increasing in c_t. Thus, if the *RHS* of (25) is less than $y_t u'(y_t) f_t'$, then there exists a unique solution for c_t with processing services. If, however, $f_t' < 1$, then the existence of a unique solution for currency balances depends on the response of u' to c_t. Define $\rho \equiv -c_t u_t'' / u_t'$ as the (quasi) Arrow-Pratt coefficient of relative risk aversion. We have the following results.

Lemma 1. If $\rho < 1/(1-f_t')$, then the *LHS* of (25) is monotonically increasing in c_t.

Proof. Let $\rho < 1/(1-f_t')$, then

$$d(LHS)/dc_t = f_t'[c_t(1-f_t')u_t'' + u_t'] > 0.$$

The lemma asserts that, as long as the household is not too risk averse, the *LHS* defines an increasing function that begins at the origin. Continuing, we have

Proposition 1. If $0 <$ *RHS* of (25) $< y_t u'(y_t) f_t'$, then there exists a unique solution for c_t with processing services.

Proof of Proposition 1. The proof follows from the Lemma and the fact that the *RHS* of (25) is a positive constant.

Equations (12) and (13) determine $(1 + r''_{t+1})^{-1}$ and q_t, respectively. $B_t = 0$, $S_t = 1$, and x_t is given by the budget constraint. Knowledge of C_t and c_t determine P_t.

The implicit function theorem can be used on (25) to verify that c_t is increasing in y_t and decreasing in $(1 + \sigma_t)$ in both sub-cases.

Now consider the relative responsiveness of currency balances to monetary and dividend shocks across regimes. The analysis is restricted to the instant at which the funds flow to the processing sector so that ρ is constant. Thus, we have the following propositions.

Proposition 2. For $f_t' > 1$, if $\rho \geq 0$ when funds initially flow to the processing sector, then relative to the no-processing services regime, real currency balances are more responsive to monetary and dividend shocks when processing services are utilized.

Proof of Proposition 2. Define $\xi \equiv RHS$ of (25) and $I \equiv c_t u'(c_t + f(y_t - c_t)) f_t' - \xi = 0$. Then,

$$I_{c_t} = f_t'[c_t(1 - f_t')u_t'' + u_t'] > 0.$$

Also,

$$I_{(1+\sigma_t)} = \xi/(1+\sigma_t) > 0$$

and

$$I_{y_t} = (f_t')^2 c_t u_t'' < 0.$$

Thus,

$$dc_t/d(1+\sigma_t)|_{k_t^d>0} = -I_{(1+\sigma_t)}/I_{c_t} < 0$$

and

$$dc_t/dy_t|_{k_t^d>0} = -I_{y_t} I_{c_t} > 0$$

From (26),

$$dc_t/d(1+\sigma_t)|_{k_t^d=0} = -\xi/u_t'(1+\sigma_t) < 0$$

and

$$dc_t/dy_t|_{k_t^d=0} = \rho > 0$$

Thus, real currency balances are more responsive to monetary and dividend shocks when processing services are utilized if

$$\frac{dc_t/d(1+\sigma_t)|_{k_t^d>0, c_t=y_t}}{dc_t/d(1+\sigma_t)|_{k_t^d=0}} = \frac{dc_t/dy_t|_{k_t^d>0, c_t=y_t}}{dc_t/dy_t|_{k_t^d=0}} = \frac{u_t'}{f_t'[c_t(1-f_t')u_t'' + u_t']} > 1.$$

This inequality is satisfied when $-(f_t')^{-1} < \rho$.

Proposition 3. For $f_t' < 1$, if $0 \leq \rho < 1/(1 - f_t')$ when funds initially flow to the processing sector, then relative to the no processing services regime, real currency balances are more responsive to monetary and dividend shocks when processing services are utilized.

Proof of Proposition 3. The proof follows from Lemma 1 and Propositions 1 and 2.

The intuitions behind these results are as follows. Consider positive dividend shocks first. A positive y_t shock has two effects. It increases the supply of goods brought to the market by sellers and the flow of funds to the processing sector. This second effect is absent in the no-processing services regime. When $f_t^{\prime} > 1$, processing services deliver consumption goods very efficiently. This additional source of pressure causes the (currency) price of goods to fall further than it would have fallen otherwise. Real currency balances rise by more than they would have in the no-processing services regime. When $f_t^{\prime} < 1$, the same effects are present; however, households must not be so risk averse that there is no unique solution for c_t.

Now consider positive monetary shocks. A positive $(1 + \sigma_t)$ shock has two effects, also. Fewer units of y_t need to be sold in order to meet any desired level of future currency holdings. Therefore, the flow of funds to the processing sector increases. This second effect is absent in the no-processing services regime. When $f_t^{\prime} > 1$, the flow of funds into the processing sector is such that the price level rises more than it would have risen otherwise. This causes real currency balances to fall by more than they would have fallen in the no-processing regime. When $f_t^{\prime} < 1$, the same effects are present; however, households must not be so risk averse that there is no unique solution for c_t.

Propositions 2 and 3 suggest that the price level is more variable in the processing services regime. The source of the additional variability is the intrasectoral substitution opportunities at households' disposal. Also note that, with respect to monetary shocks, these results are contrary to the usual findings in Svensson CIA models. In those models, real balances do not respond to monetary shocks once the CIA constraint binds.

Appendix B

Dividend Income Shocks

Figure 5 shows the responses of the model's endogenous variables to a one standard deviation shock to dividend income. On impact, the increase in dividends causes the price level to fall. This raises inflation going into the next period. The increase in the nominal interest rate is positive but negligible. The reason is that the impact of increased inflation on nominal interest rates is largely offset by the anticipated decline in dividend income. The anticipated decline in future dividend income is met with an anticipated decline in consumption and the demand for currency that leaves velocity practically unchanged. The anticipated increased willingness to lend currency in the future puts downward pressure on the nominal interest rate. The value of the firm rises in response to the positive news about its current and future dividends.

FIGURE 5A. RESPONSE TO A DIVIDEND SHOCK

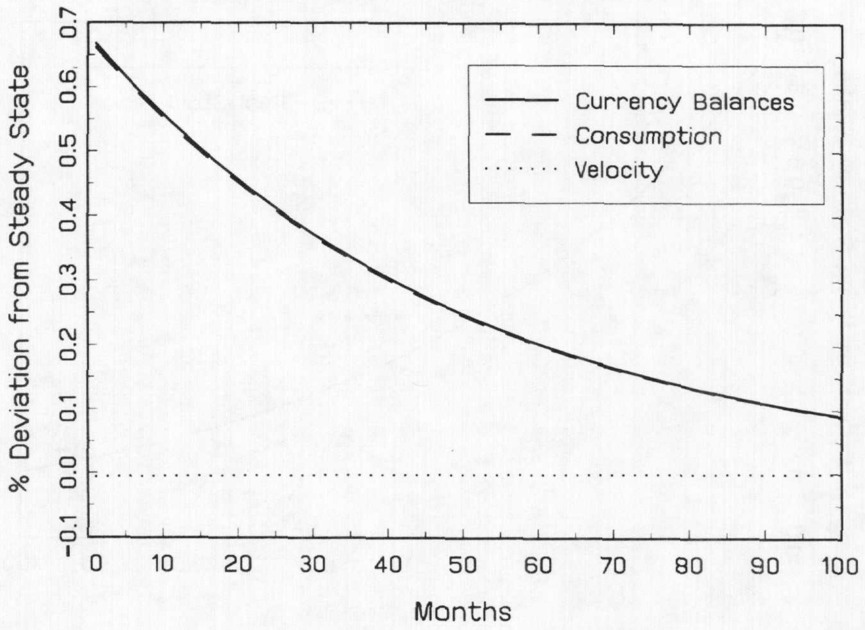


FIGURE 5B. RESPONSE TO A DIVIDEND SHOCK

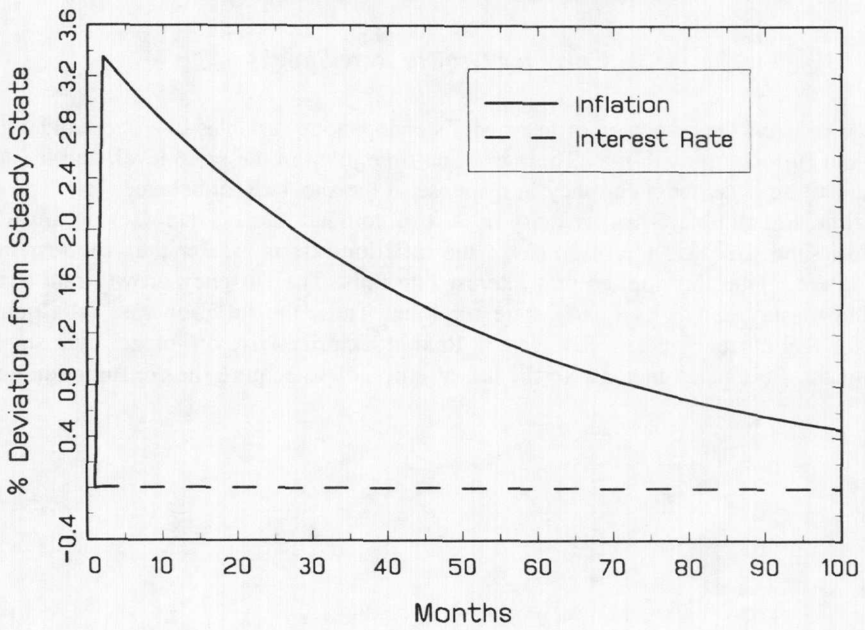
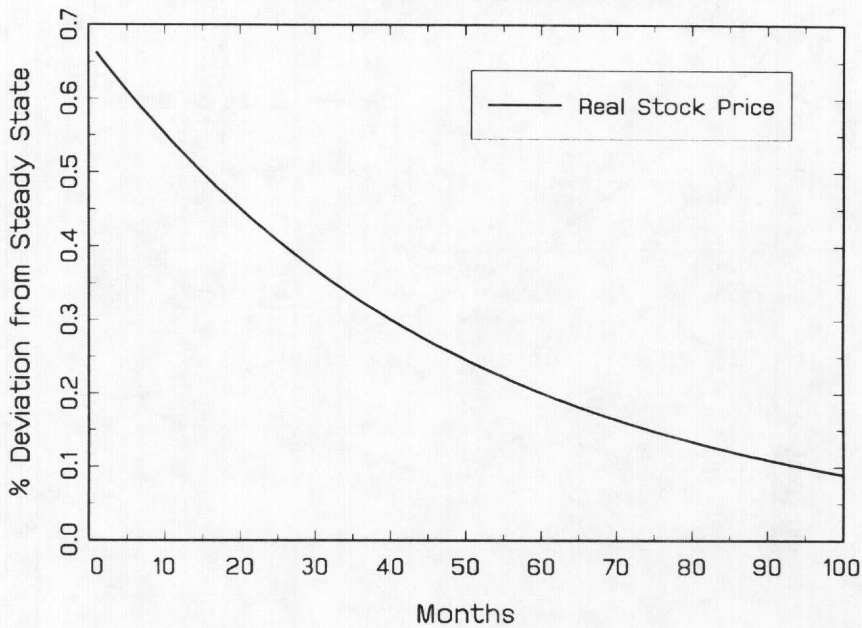


FIGURE 5C. RESPONSE TO A DIVIDEND SHOCK



Currency Growth Rate Shocks

Figure 6 shows the responses of the model's endogenous variables to a one standard deviation shock to the currency growth rate. There is a one-time jump in the price level, and inflation rises. Agents substitute away from currency in response to the shock. Real balances fall. The change in consumption is negligible. Thus, velocity rises. The nominal interest rate rises on impact as more bonds are issued. Because the response of the inflation rate is greater than the response of the nominal interest rate, the implied real interest rate falls. The currency growth rate shock is not anticipated to stay above its steady state for long. Thus, the inflation rate falls quickly. The demand for real currency balances recovers. Real stock prices rise on impact by a small amount because of the anticipated increase in the use of currency to acquire the consumption good in the future.

FIGURE 6A. RESPONSE TO A CURRENCY GROWTH SHOCK

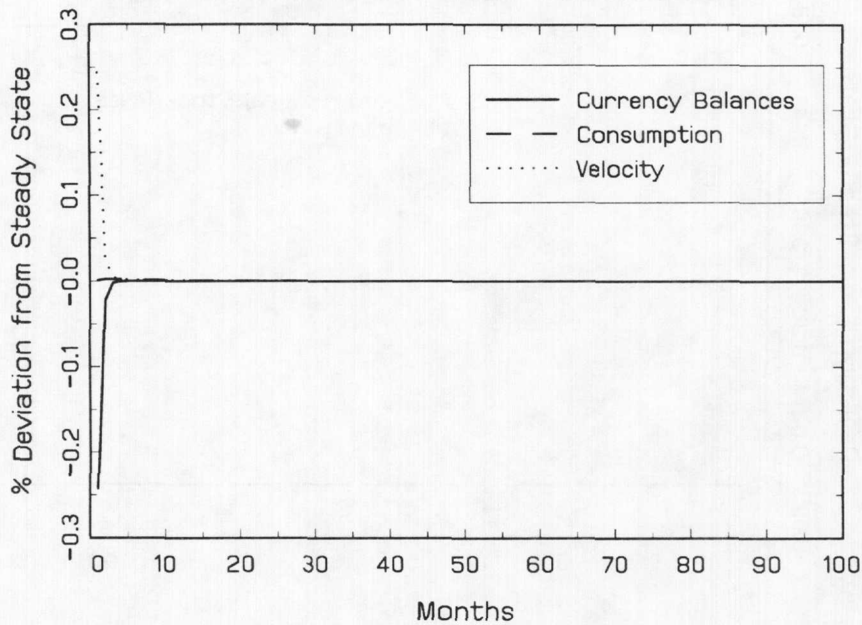


FIGURE 6B. RESPONSE TO A CURRENCY GROWTH SHOCK

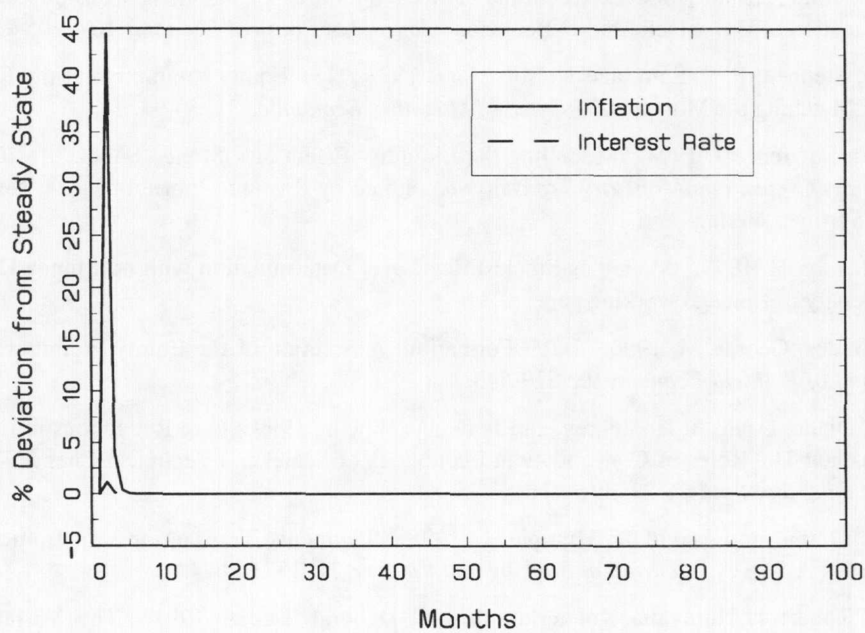
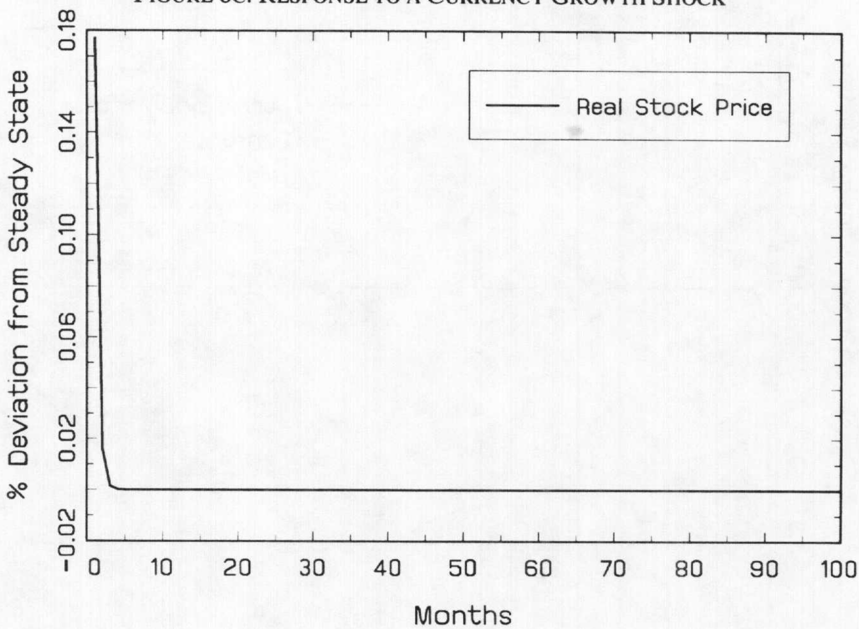


FIGURE 6C. RESPONSE TO A CURRENCY GROWTH SHOCK



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